Here’s my solutions to the medium exercises in chapter 2 of McElreath’s Statistical Rethinking, 1st edition. My intention is to move over to the 1nd edition when it comes out next month.

\(\DeclareMathOperator{\dbinomial}{Binomial} \DeclareMathOperator{\dbernoulli}{Bernoulli} \DeclareMathOperator{\dpoisson}{Poisson} \DeclareMathOperator{\dnormal}{Normal} \DeclareMathOperator{\dt}{t} \DeclareMathOperator{\dcauchy}{Cauchy} \DeclareMathOperator{\dexponential}{Exp} \DeclareMathOperator{\duniform}{Uniform} \DeclareMathOperator{\dgamma}{Gamma} \DeclareMathOperator{\dinvpamma}{Invpamma} \DeclareMathOperator{\invlogit}{InvLogit} \DeclareMathOperator{\logit}{Logit} \DeclareMathOperator{\ddirichlet}{Dirichlet} \DeclareMathOperator{\dbeta}{Beta}\)

**Globe Tossing**

Start by creating a grid and the function posterior which we we use for several calculations. This is analogous to the code provided in the chapter.

p\_true <- 0.7 # assumed ground truth

granularity <- 1000 # number of points on grid

grid1 <- tibble(p = seq(0, 1, length.out = granularity)) %>%

mutate(prior = 1)

posterior <- function(data, grid) {

grid %>%

mutate(

likelihood = dbinom(sum(data == 'W'), length(data), p),

unstd\_posterior = prior \* likelihood,

posterior = unstd\_posterior / sum(unstd\_posterior)

)

}

The exercise asks us to approximate the posterior for each of the following three datasets. To do this, we just apply our posterior function above to each of them.

data <- list(

'1' = c('W', 'W', 'L'),

'2' = c('W', 'W', 'W', 'L'),

'3' = c('L', 'W', 'W', 'L', 'W', 'W', 'W')

)

m1 <- data %>%

map\_dfr(posterior, grid1, .id = 'dataset')

Solution 2M1

The posterior becomes gradually more concentrated around the ground truth.

For the second question, we simply do the same but with a different prior. More specifically, for any p below 0.5 we set the prior to zero, then map our posterior over each the the datasets with this new grid.

grid2 <- grid1 %>%

mutate(prior = if\_else(p < 0.5, 0, prior))

m2 <- data %>%

map\_dfr(posterior, grid2, .id = 'dataset')

Solution 2M2

Again we see the posterior concentrate more around the ground truth. Moreover, the distribution is more peaked (at ~ 0.003) than with the uniform prior, which peaks at around (~0.0025). The first dataset already gets pretty close to this peak, i.e. this more informative prior gets us better inferences sooner.

For the final question on globe tossing, we can just use the counting method rather than grid approximation. We enumerate all possible events in proportion to how likely they are to occur: 10 L for Mars, 3 L and 7 W for Earth. Then we filter our any inconsistent with our observation of land, and summarise the remaining possibilities.

m3 <- tibble(mars = rep('L', 10)) %>%

mutate(earth = if\_else(row\_number() <= 3, 'L', 'W')) %>%

gather(planet, observation) %>% # all possible events

filter(observation == 'L') %>% # only those events consistent with observation

summarise(mean(planet == 'earth')) %>% # fraction of possible events that are earth

pull()

m3

[1] 0.2307692

We get around 23%.

**Card Drawing**

We make a list of all sides, filter out any inconsistent with our observation of a black side, then summarise the remaining card possibilities.

m4\_events <- tibble(card = c("BB", "BW", "WW")) %>% # all the cards

separate(card, into = c('side1', 'side2'), sep = 1, remove = F) %>%

gather(side, colour, -card) # all the sides

m4\_possibilities <- m4\_events %>%

filter(colour == 'B') # just the possible events where there is a black side

m4 <- m4\_possibilities %>%

summarise(mean(card == 'BB')) %>%

pull() # which fraction of possible events is a double black?

m4

[1] 0.6666667

The next exercise is the same as the previous but with more cards. Note that this equivalent to using the three cards as before but with a larger prior probability on the BB card.

m5\_events <- tibble(card = c("BB", "BW", "WW", "BB")) %>%

separate(card, into = c('side1', 'side2'), sep = 1, remove = F) %>%

gather(side, colour, -card)

m5\_possibilities <- m5\_events %>%

filter(colour == 'B')

m5 <- m5\_possibilities %>%

summarise(mean(card == 'BB')) %>%

pull()

m5

[1] 0.8

Putting the prior on the cards is equivalent to having the cards in proportion to their prior. The rest of the calculation is the same.

m6\_events <- c("BB", "BW", "WW") %>% # cards

rep(c(1, 2, 3)) %>% # prior: repeat each card the given number of times

tibble(card = .) %>%

separate(card, into = c('side1', 'side2'), sep = 1, remove = F) %>%

gather(side, colour, -card)

m6\_possibilities <- m6\_events %>% # sides

filter(colour == 'B')

m6 <- m6\_possibilities %>% # sides consistent with observation

summarise(mean(card == 'BB')) %>% # proportion of possible events that are BB

pull()

m6

[1] 0.5

This last card drawing exercise is slightly more involved since we can observe any of the two sides of the one card and any of the two sides of the other. Thus, we first generate the list of all possible pairs of cards, expand this into a list of all possible sides that could be observed for each card, filter out any event not consisent with our observations, then summarise whatever is left.

m7\_card\_pairs <- tibble(card = c("BB", "BW", "WW")) %>% # all the cards

crossing(., other\_card = .$card) %>%

filter(card != other\_card) # all card pairs (can't draw the same card twice)

m7\_events <- m7\_card\_pairs %>%

separate(card, into = c('side1', 'side2'), sep = 1, remove = F) %>%

separate(other\_card, into = c('other\_side1', 'other\_side2'), sep = 1, remove = F) %>%

gather(side, colour, side1, side2) %>% # all the sides for card of interest

gather(other\_side, other\_colour, other\_side1, other\_side2) # all sides of other card

m7\_possibilities <- m7\_events %>%

filter(

colour == 'B', # we observe that card of interest has a black side

other\_colour == 'W' # we observe that the other card has a white side

)

m7 <- m7\_possibilities %>%

summarise(mean(card == 'BB')) %>% # which fraction of possible events is a double black?

pull()

m7

[1] 0.75